QAT Sample Questions (SET 4)

1. Let $f(x)$ be a function that is differentiable up to order 3, and let

$$f(-1) = -6;\ f(0.5) = 21;\ f'(-1) = 3.$$

Assuming that $f'''(x)$ is very close to 0 for all x, what is $f''(-1)$ approximately?

A. 1
B. 10
C. 12
D. 15
E. 20

2. Which of the following is equal to $\int_0^1 \int_0^y f(x,y)\, dx\, dy$

A. $\int_0^1 \int_x^1 f(x,y)\, dy\, dx$
B. $\int_0^1 \int_0^1 f(x,y)\, dy\, dx$
C. $\int_0^1 \int_0^x f(x,y)\, dy\, dx$
D. $\int_0^x \int_0^1 f(x,y)\, dx\, dy$
E. $\int_0^1 \int_y^1 f(x,y)\, dx\, dy$

3. Which of the following functions has a local minimum but no global minimum? (The domain of f is R and the domain of g is $R \times R$.)

A. $f(x) = x^2 - 2x + 1$
B. $f(x) = (x+1)(x-2)(x-3)$
C. $f(x) = \sin x$
D. $g(x,y) = x^2 + y^2$
E. $g(x,y) = e^{x+y}$

4. Using substitution, which of the following equations are solutions to the partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$

   A. cos(3x-y)

   B. $x^2 + y^2$

   C. sin(3x-3y)

   D. $\exp^{-3\pi x}\sin(\pi y)$

   E. 5-x

5. Suppose $\frac{dy}{dt} = 5 - x$ and y=8 when x=2. Use Euler's method with increments of Δx=-0.1 to approximate the value of y when x=1.7

   A. 7.07

   B. 7.05

   C. 7.5

   D. 7.07

   E. 7.08

6. A tank initially holds 100 litres of a brine solution containing 20 kg of salt. At t=0, fresh water is poured into the tank at the rate of 5 kg/minute, while the well stirred mixture leaves the tank at the same rate. Find the amount of salt at any time t.

   A. $10\exp^{(t)}$

   B. $20\exp^{(-t/20)}$

   C. 22cos(t/20)

   D. $2\exp^{(t/20)}$

   E. 22

7. In a year, a tech company records its sales in a 4×3 matrix A. The values in the ith row of A represent the sales in the ith quarter of smartphones, tablets, and laptops, respectively. Which of the following matrices shows the *cumulative* sales at the end of each quarter for each product?

A. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} A$

B. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} A$

C. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} A$

D. $A \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

E. $A \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

8. Let v be a column vector and I the identity matrix.

$(I + vv^T)^{-1} =$

A. $I + \dfrac{vv^T}{1 + v^T v}$

B. $I - \dfrac{vv^T}{1 + v^T v}$

C. $\dfrac{I + vv^T}{1 + v^T v}$

D. $\dfrac{I - vv^T}{1 + v^T v}$

E. $\dfrac{vv^T}{1 + v^T v}$

9. Suppose that U, V, W are linearly independent vectors. Consider the following statements:
   i. $U + 2V,\ V + 2W,\ W + 2U$ are linearly independent vectors
   ii. $U - V,\ V - W,\ W - U$ are linearly independent vectors
   iii. $U - W,\ V + W$ are linearly independent vectors

   Which statements are true?

   A. i. only
   B. i. and ii.
   C. i. and iii.
   D. ii. and iii.
   E. None

10. Two coupons are randomly chosen from a box that has ten coupons numbered 1, 2, …, 10. What is the probability that the numbers add up to 8?

    A. 1/15
    B. 2/15
    C. 2/45
    D. 4/45
    E. 8/45

11. Adam toss a coin 5 times independently. Each time the coin lands on heads with probability 37/252. If he gets 2 heads from the 5 tosses, what is the probability that the first toss is a head?

    A. 37/252
    B. 37/126
    C. 1/5
    D. 2/5
    E. 1/2

12. An urn consists of one white balls and one red ball. Rob and Tom alternately randomly choose a ball. Each time a white ball is chosen, it will be placed back in the urn, and an additional white ball is added into the urn. If a red ball is chosen, it will be removed from the urn. Let

$$X = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}.$$

    If Rob starts first, what is the probability that Rob is the person who gets the red ball in terms of X?

    A. $X/2$
    B. X
    C. $2X$
    D. X^2
    E. 1/2

Use the following information to answer Questions 13 - 15.

Let X and Y be random variables with the following expected values, variances, and covariance:

$$E[\ln(X)] = 3, E[\ln(Y)] = 1.5, Var[\ln(X)] = 4, Var[\ln(Y)] = 1.44, \text{ and } Cov[\ln(X), \ln(Y)] = 1.08,$$

where $\ln(.)$ denotes the natural log function, $\ln(2) = 0.693$, and $\ln(3) = 1.097$. Suppose that $x_1, \ldots, x_n$ is a random sample of size n, independently drawn from the distribution of X. Define

$$\overline{\ln(X)}_n = \frac{1}{n}\sum_{i=1}^{n} \ln(x_i);$$

$$\hat{\sigma}^2_{\ln(X),n} = \frac{1}{n-1}\sum_{i=1}^{n} \left(\ln(x_i) - \overline{\ln(X)}_n\right)^2.$$

Let $z \sim N(0,1)$. Then, $P\{z \leq 0.082\} = 0.533$, $P\{z \leq 0.165\} = 0.566$, $P\{z \leq 0.823\} = 0.795$, $P\{z \leq 1.645\} = 0.950$, $P\{z \leq 1.960\} = 0.975$, $P\{z \leq 2.326\} = 0.990$, and $P\{z \leq 2.576\} = 0.995$.

13. Which of the following choices is incorrect if $\ln(X)$ and $\ln(Y)$ is Normally distributed?
    A. $E[X] < E[Y^3]$
    B. $E[\ln(2X)] < E[\ln(X^2)]$
    C. $E[\ln(2X)] < E[(\ln(X))^2]$
    D. $Cov[\ln(X), \ln(Y)] < Cov[\ln(X), \ln(Y^2)]$
    E. $Corr[\ln(X), \ln(Y)] < Corr[\ln(X), \ln(Y^2)]$

14. What is the value of covariance between $\ln(3X^2)$ and $\ln(Y)$?
    A. 2.16
    B. 4.32
    C. 2.16 + $\ln(3)$
    D. 4.32 + $\ln(3)$
    E. 4.32 $\times$ $\ln(3)$

15. What is the mean or expected value of $\ln\left(\frac{3X^2}{Y}\right)$?
    A. $\ln(3) \times 6$
    B. $\ln(3) \times 4.5$
    C. $\ln(3) \times 7.5$
    D. $\ln(3) + 4.5$
    E. $\ln(3) + 7.5$

List of symbols for Questions 16 - 18

Symbol	Description
$\sim$	Negation
$\wedge$	Logical 'and'
$\vee$	Logical 'or'
$P \Longrightarrow Q$	P is sufficient for Q. Q is necessary for P. If P then Q. Q only if P
$P \Longleftrightarrow Q$	P is equivalent to Q
$\forall x$	For all x
$\exists x$	There exists an x

16. What must be done to prove the statement below?

"e is at least 2.7 and e is not more than 2.8"

A. Prove that $e \geq 2.7$
B. Prove that either $e > 2.7$ or $e < 2.8$
C. Prove that either $e > 2.7$ or $e \leq 2.8$
D. Prove that $e > 2.7$ and $e < 2.8$
E. Prove that $e \geq 2.7$ and $e \leq 2.8$

17. Given the equation:

$$r(0,2) = \frac{1}{2}r(0,1) + \frac{1}{2}E\big(r(1,2)\big) + \Pi$$

and suppose that $r(0,2) > r(0,1)$. Consider the following statements:

P1: If $\Pi \leq 0$ then $\mathrm{E}\big(\mathrm{r}(1,2)\big)$ must be greater than $\mathrm{r}(0,1)$

P2: If $\Pi > 0$ then $\mathrm{E}\big(\mathrm{r}(1,2)\big)$ must be less than $\mathrm{r}(0,1)$

Which of the follows is correct?

A. Both P1 and P2 are true
B. Both P1 and P2 are false
C. P1 is true while P2 is false
D. P1 is false while P2 is true
E. There is not enough information to decide the truth value of both statements

18. Given the following statement:

All birds can fly

Let x be an animal. Let $P(x)$ means x is a bird, and $Q(x)$ means x can fly. Which of the follows is equivalent to the above statement?

A. $\forall x[P(x) \Longrightarrow Q(x)]$
B. $\forall x[P(x) \wedge Q(x)]$
C. $\nexists x[\sim P(x) \wedge \sim Q(x)]$
D. $\nexists x[\sim P(x) \vee \sim Q(x)]$
E. There is no equivalent statement

19. In recent years, Chiangmai has taken to hosting a Songkran festival, a celebration of Thailand's heritage that reflects on its primitive past while exalting its pop culture driven present. But clearly only children take this festival seriously, for they are the only attendees who bother to dress up in Thai traditional costume.
Which of the following is an assumption on which the argument depends?

A. Any attendee who dresses up in a Thai traditional costume takes the festival seriously.
B. Anyone who is not dressed up in a Thai traditional costume is not attending the festival.
C. No attendee who takes the festival seriously would fail to dress up in a Thai traditional costume.
D. The festival organizers have instituted a Thai traditional costume dress code.
E. If an attendee is not dressed in a Thai traditional costume, then that attendee will not be taken seriously by other attendees.

20. The percentage of households with an annual income of more than 600,000 Baht is higher in Phuket than in any other provinces. However, the percentage of households with an annual income of 800,000 Baht or more is higher in Bangkok.
If the statements above are true, which of the following must also be true?

A. The percentage of households with an annual income of 1,000,000 Baht is higher in Bangkok than in Phuket.
B. Phuket has the second highest percentage of households with an annual income of 800,000 Baht or more.
C. Average annual household income is higher in Bangkok than in Phuket.
D. Some households in Phuket have an annual income between 600,000 and 800,000 Baht.
E. The number of households with an annual income of more than 600,000 Baht is greater in Phuket than in Bangkok.

21. In Thailand, of the people who moved from one province to another when they retired, the percentage who retired to Chiangrai has decreased by three percentage points over the past five years. Since many local businesses in Chiangrai cater to retirees, these declines are likely to have a noticeably negative economic effect on these businesses and therefore on the economy of Chiangrai.
Which of the following, if true, most seriously weakens the argument given?

A. People were more likely to retire to Phuket from another province last year than people were five years ago.
B. The number of people who left Chiangrai when they retired to live in another province was greater last year than it was five years ago.
C. People who moved from one state to another when they retired moved a greater distance, on average, last year than such people did five years ago.
D. Chiangrai attracts more people who move from one province to another when they retire than does any other provinces.
E. The number of people who moved from one province to another when they retired has increased significantly over the past five years.

Solutions:

1. E
2. A
3. B
4. A
5. D
6. B
7. A
8. B
9. C
10. A
11. D
12. B
13. E
14. A
15. D
16. E
17. C
18. A
19. C
20. D
21. E